Snow College Jr. Mathematics Contest

key

April 3, 2012 **Junior Division: Grades 7–9** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Evaluate the expression.

$$\frac{\sqrt{36}+\sqrt{64}}{\sqrt{36+64}}$$

(A) 1/5

[(B)] 7/5

(C) 16/5

(D) 24/5

(E) 1

SOLN $\frac{6+8}{10}=\frac{7}{5}$ □

2. What is the sum of the units digits of all the multiples of 3 between 0 and 50?

(A) 75

(B) 77

[(C)] 78

(D) 80

(E) 82

SOLN $3+6+9+2+5+8+1+4+7+0+3+6+9+2+5+8=78$ □

3. In shopping for a new calculator, Sue found an \$80 graphing calculator which was on sale for 15% off. How much did she pay?

(A) \$52

(B) \$56

(C) \$60

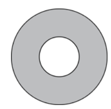
(D) \$64

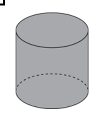
[(E)] \$68

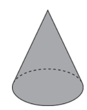
SOLN $80(1-0.15)=80(0.85)=68$ □

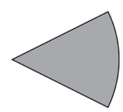
4. Which shaded region fits the area formula?

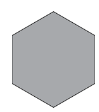
$$A=\pi(R^2-r^2)$$

[(A)] 

(B) 

(C) 

(D) 

(E) 

SOLN The area of the outer circle is πR^2; the area of the inner is πr^2. □

5. What is the measure of the acute angle between the hour and minute hands of a correctly working analog clock at 4:18?

(A) $12°$

(B) $15°$

(C) $18°$

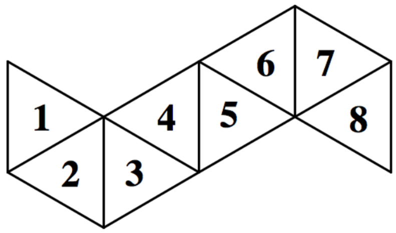
[(D)] $21°$

(E) $24°$

SOLN After 18 minutes the minute hand has gone through an angle of $\frac{18}{60}(360°)=108°$. The hour hand has gone through $\frac{4}{12}(360°)+\frac{18}{60}\left(\frac{360°}{12}\right)=120°+9°=129°$. □

6. A regular octahedron is folded from the net shown. What number shows on the top when the face numbered 1 is on bottom?

(A) 3
(B) 5
(C) 6
(D) 7
[(E)] 8



SOLN 2–7 wrap around to be the sides, leaving 1 and 8 for top and bottom. □

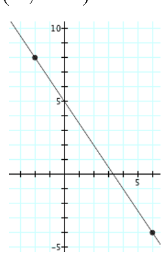
7. A farmer plans to build a fence around a rectangular pasture. He wants to place a post every 10 ft. If the pasture is 240 ft × 320 ft, how many posts will he need?

(A) 55
(B) 56
(C) 111
[(D)] 112
(E) 116

SOLN Count corner posts only once: 24 posts on a short side and 32 on a long side. $\frac{\text{perim.}}{10\,\text{ft}} = 112$ □

8. Find the equation of the line that goes through the points $(-2, 8)$ and $(6, -4)$.

[(A)] $3x + 2y = 10$
(B) $3x + 2y = 6$
(C) $-3x + 2y = -6$
(D) $-3x + 2y = -26$
(E) $3x + 2y = 26$



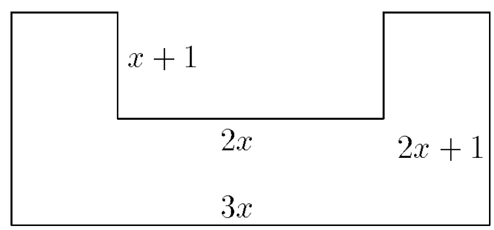
SOLN $m = \frac{8-(-4)}{-2-6} = \frac{12}{-8} = \frac{-3}{2}$
$y - (-4) = \frac{-3}{2}(x - 6)$
$y = \frac{-3}{2}x + 5 \implies 2y = -3x + 10$ □

9. How many natural number factors does 2012 have?

(A) 3
(B) 4
(C) 5
[(D)] 6
(E) 12

SOLN The prime factorization of 2012 is $2^2 \cdot 503$. So the factors are 1, 2, 4, 503, 1006, 2012. □

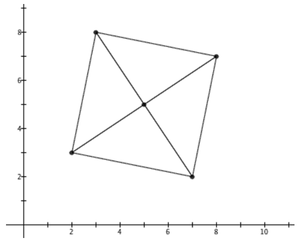
10. Write an expression for the area.



(A) $(2x + 1)(x + 1)$
(B) $6x(x + 1)$
(C) $4x(x + 1)$
(D) $4x(2x + 1)$
[(E)] $4x^2 + x$

SOLN The bottom rectangle has area $(3x)[(2x + 1) - (x + 1)] = 3x^2$. The two upper pieces together have height $(x + 1)$ and width $3x - 2x$ so the area of those pieces is $(x + 1)x$. Alt. Soln: area of region plus hole is $3x(2x + 1)$ and the area of the hole is $2x(x + 1)$, so the area of the region is $6x^2 + 3x - (2x^2 + 2x)$. □

11. If $(3, 8)$ and $(7, 2)$ are the coordinates of two opposite vertices of a square, what are the coordinates of the other two vertices?



(A) $(3, 7)$, $(2, 8)$

(B) $(3, 2)$, $(7, 8)$

[(C)] $(2, 3)$, $(8, 7)$

(D) $(2, 7)$, $(8, 3)$

(E) $(3, 3)$, $(8, 8)$

[SOLN] The given vertices lie on one of the diagonals. Since the slope of this diagonal is $-\frac{3}{2}$, the slope of the other diagonal must be $\frac{2}{3}$. The midpoint of both diagonals is $(5, 5)$. □

12. Find the 100th digit after the decimal point in $0.\overline{341729}$.

(A) 1

(B) 2

(C) 3

(D) 4

[(E)] 7

[SOLN] 6 divides into 100 16 times with remainder 4, so the 100th digit is the 4th digit of the repeating pattern. □

13. Uncle Bookworm eats two books a week; Aunt Bookworm eats one book every two months. In a year, how many more books does Uncle eat than Aunt?

(A) 17

(B) 20

(C) 40

(D) 80

[(E)] 98

[SOLN] Uncle: $2\,\frac{\text{bks}}{\text{wk}}\left(\frac{52\text{ wks}}{1\text{ year}}\right) = 104\,\frac{\text{books}}{\text{year}}$

Aunt: $1\,\frac{\text{book}}{2\text{ mo}}\left(\frac{12\text{ mo}}{1\text{ year}}\right) = 6\,\frac{\text{books}}{\text{year}}$ □

14. Elaine and Dan want to install wall-to-wall carpeting in their family room. The floor of the rectangular room is 18 feet long and $13\frac{3}{5}$ feet wide. How much will it cost to carpet the room if carpet costs $\$30/\text{yd}^2$?

[(A)] \$816

(B) \$1237

(C) \$2171

(D) \$6514

(E) \$58,627

[SOLN] $(18\,\text{ft})\left(\frac{1\,\text{yd}}{3\,\text{ft}}\right) = 6\,\text{yd}$

$13\frac{3}{5}\,\text{ft} = \left(\frac{68}{5}\,\text{ft}\right)\left(\frac{1\,\text{yd}}{3\,\text{ft}}\right) = \frac{68}{15}\,\text{yd}$

$A = L \times W = (6\,\text{yd})\left(\frac{68}{15}\,\text{yd}\right) = \frac{136}{5}\,\text{yd}^2$

$\text{Cost} = \left(\frac{136}{5}\,\text{yd}^2\right)\left(\frac{\$30}{\text{yd}^2}\right) = \$816$ □

15. If my pet runs 300 cm/s and your rocket flies 300 m/s then how many times as fast is your rocket as my pet?

(A) 30000

(B) 1000

(C) 300

[(D)] 100

(E) 1

[SOLN] $300\,\text{m/s} = 30000\,\text{cm/s}$ which is $100\times$ as fast as $300\,\text{cm/s}$. □

16. Find the probability of randomly choosing a dark caramel from a box that contains only light and dark caramels and has twice as many light as dark caramels.

(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{1}{4}$

[(E)] $\frac{1}{3}$

[SOLN] If there are d dark caramels, then there are $2d$ light caramels. The total number of caramels is $d + 2d = 3d$. So the probability is $\frac{d}{3d} = \frac{1}{3}$. □

17. My pet rabbit, Cotton, can hop up one step at a time or two steps at a time. The stairs in my house have ten steps. How many ways can Cotton get up my stairs?

(A) 20

(B) 32

[(C)] $89 = 1 + 15 + 35 + 28 + 9 + 1$

(D) 117

(E) 1024

[SOLN] Note pattern in short staircases.

steps	ways	#
1	1	1
2	11, 2	2
3	111, 12, 21	3
4	1111, 112, 121, 211, 22	5
5	11111, 1112, 1121, 1211, 2111, 221, 212, 122	8

The pattern is the Fibonacci sequence; the 10th number is 89.

Alt. Soln: tally the combinations to climb 10 stairs; count ways for each.

$10 = 5 \cdot 2 + 0 \cdot 1 \rightarrow \binom{5}{0} = 1$

$10 = 4 \cdot 2 + 2 \cdot 1 \rightarrow \binom{6}{2} = 15$

$10 = 3 \cdot 2 + 4 \cdot 1 \rightarrow \binom{7}{4} = 35$

$10 = 2 \cdot 2 + 6 \cdot 1 \rightarrow \binom{8}{6} = 28$

$10 = 1 \cdot 2 + 8 \cdot 1 \rightarrow \binom{9}{8} = 9$

$10 = 0 \cdot 2 + 10 \cdot 1 \rightarrow \binom{10}{10} = 1$ □

18. The sum of two numbers is 10; their product is 20. Find the sum of their reciprocals.

(A) $\frac{1}{10}$

[(B)] $\frac{1}{2}$

(C) 1

(D) 2

(E) 4

[SOLN] Long way: equations $x + y = 10$ and $xy = 20$ lead to the quadratic formula and rationalizing a denominator.

Short way: $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{10}{20}$ □

19. Say you place a 25 000-mile-long metal band snugly around the earth's equator. (Assume a smooth spherical earth.) Then you cut the band and splice another 50 feet to it, thus loosening it all around. What is the tallest object that could comfortably fit between the new-length band and the earth?

(A) a DNA molecule

(B) a grain of sand

(C) a golf ball

(D) a small dog

[(E)] a tall person

[SOLN] The long way is to find the radius of a 25 000-mile-circumference circle and again for a circle of circumference 25 000 miles plus 50 feet.

The short way is to make a straight-line graph of circumference vs. radius for circles.

$$2\pi = \frac{\text{rise}}{\text{run}} = \frac{\Delta C}{\Delta r} \implies$$

$$\Delta r = \frac{\Delta C}{2\pi} = \frac{50\,\text{ft}}{2\pi} \approx 8\,\text{ft}$$

Original circumference is irrelevant. From Marilyn vos Savant's Parade Magazine column on 12 June 2011. □

20. When 15 is added to a set of ten numbers, the median changes from 6 to 8. Find the median of the new set if 7 replaces 15.

(A) 4

(B) 5

(C) $5\frac{1}{2}$

(D) 6

[(E)] 7

[SOLN] Originally, the median, 6, is the average of the 5th and 6th numbers, and the 6th must be 8 (because it is the new median when 15 is added). Replacing 15 with 7 puts the 7 in the middle location (6th). □