# Snow College Mathematics Contest

$\boxed{\text{key}}$

**March 18, 2025** **Senior Division: Grades 10-12** **Form:** $\boxed{\text{T}}$

Bubble in clearly the single best choice for each question you choose to answer.

1. Kim plays basketball for her school. Her free-throw shooting percentage for the season was 75% exactly before today. During tonight's game she makes all five free throws, bringing her percentage up to 80%. How many free throws has Kim made in the season (including tonight)?

$\boxed{\text{(A)}}$ 20
(B) 22
(C) 24
(D) 25
(E) 28

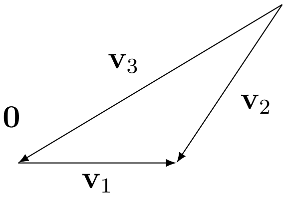
$\boxed{\text{SOLN}}$ Call her shots taken before tonight $x$. Her shots made can be represented as

$$\tfrac{3}{4}x + 5 = \tfrac{4}{5}(x+5)$$

where the left side is shots made before tonight + 5, and the right side is 80% of all shots made. Solving for $x$ gives $x = 20$; then the left side becomes $15 + 5 = 20$. ◻

2. A vector is a quantity with both size and direction; they are often represented by arrows. Vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ are shown below. Which of the following is the correct relationship between the three vectors?

(A) $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_3$
(B) $\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{v}_3$
(C) $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$
(D) $\mathbf{v}_3 - \mathbf{v}_2 = \mathbf{v}_1$
$\boxed{\text{(E)}}$ $\mathbf{v}_1 + \mathbf{v}_3 = \mathbf{v}_2$



$\boxed{\text{SOLN}}$ Geometrically, vectors are added by placing a pair tip to tail as $\mathbf{v}_1$ and $\mathbf{v}_3$ are shown. The resultant vector is then drawn from the tail of the first to the tip of the second. ◻

3. Professor Spinner is playing basketball with her son, Fidget. Fidget is fouled and gets two foul shots. The probability that Fidget makes the first shot is $\frac{3}{4}$. If he makes the first shot, the probability that he makes the second shot is $\frac{9}{10}$. If he misses the first shot, the probability that he makes the second shot is $\frac{1}{2}$. What is the probability that Fidget makes at least one shot?

(A) $\frac{3}{40}$
(B) $\frac{1}{8}$
(C) $\frac{5}{8}$
(D) $\frac{27}{40}$
$\boxed{\text{(E)}}$ $\frac{7}{8}$

$\boxed{\text{SOLN}}$ $P(\text{makes at least one shot}) = 1 - P(\text{misses both shots}) = 1 - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)$. Also: $\frac{3}{4} + \frac{1}{4} \cdot \frac{1}{2} = \frac{7}{8}$ ◻

4. Mr. Pierce has 10 students show up for his class service project on Earth Day. He needs to assign 7 of the students to plant trees around the school. The remaining 3 students will plant trees at the public library. How many different ways can Mr. Pierce assign his 10 students to these two tree-planting locations?

$\boxed{\text{(A)}}$ 120
(B) 240
(C) 720
(D) 13400
(E) 604800

$\boxed{\text{SOLN}}$

$$\binom{10}{7}\binom{3}{3} = \frac{10!}{7!\,3!}\,\frac{3!}{3!\,0!} = (120)(1)$$ ◻

5. If a $k^{\text{th}}$ power *generalized mean* is defined as

$$M_k = \left(\frac{a^k + b^k}{2}\right)^{1/k}$$

we get the RMS (root-mean-square) value for $k = 2$, the geometric mean for $k \to 0$, and the harmonic mean for $k = -1$. Which value of $k$ produces the arithmetic mean?

(A) $-2$

(B) $0.5$

$\boxed{\text{(C)}}$ $1$

(D) $3$

(E) $4$

$\boxed{\mathcal{SOLN}}$ Trial and error shows that $k = 1$ gives $M_1 = \frac{a+b}{2} = \text{AM}$.

$$\text{Note:} \qquad \text{RMS} \geq \text{AM} \geq \text{GM} \geq \text{HM}$$

The $k = 0$ case is very interesting, but by taking the limit $k \to 0$ we get $M_0 = \sqrt{ab}$. `https://aapt.scitation.org/doi/full/10.1119/10.0000841`. □

6. Consider the points $A(-5, 1)$ and $C(3, -3)$ shown below. A point $B(b, 0)$ is placed along the $x$-axis. The shortest distance from $A$ to $C$ through $B$ occurs if $B$ is on the line containing $A$ and $C$. But what value of $b$ will give the minimum of $|AB|^2 + |BC|^2$?

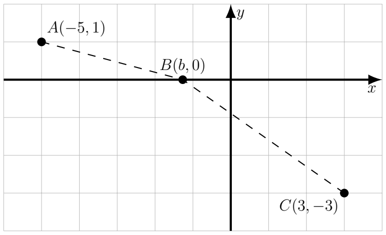
(A) $-5$

(B) $-3$

(C) $-2$

$\boxed{\text{(D)}}$ $-1$

(E) $0$



$\boxed{\mathcal{SOLN}}$ The function describing this distance is $d = (b-(-5))^2 + (0-1)^2 + (3-b)^2 + (-3-0)^2 = 2b^2 + 4b + 44$. The vertex of this parabola is a minimum and it occurs at $\frac{-4}{2\cdot 2}$. Alternatively, the derivative of this is $d' = 4b + 4$ which gives a critical value and a minimum for $d$ at $b = -1$. □

7. Find the total area between $y = \sin(x)$ and $y = \cos(x)$ on the interval $[0, \pi]$.

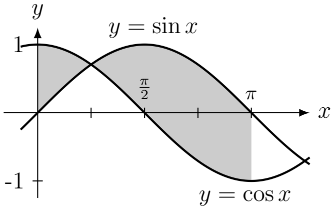
$\boxed{\text{(A)}}$ $2\sqrt{2}$

(B) $2$

(C) $5\pi/6$

(D) $\pi - 1/4$

(E) $-2$



$\boxed{\mathcal{SOLN}}$

$$\int_0^{\pi/4} \cos(x) - \sin(x)\,dx + \int_{\pi/4}^{\pi} \sin(x) - \cos(x)\,dx = 2\sqrt{2}$$ □

8. One extension of the real numbers is the complex numbers $z = a + b\mathrm{i}$, where $a, b$ are real numbers and $\mathrm{i}^2 = -1$. ($\bar{z} = a - b\mathrm{i}$.) Another is the dual numbers $z = a + b\varepsilon$, where $a, b$ are real numbers and $\varepsilon^2 = 0$. A "unit circle" is the locus of points $z$ such that $|z| = 1$, or, equivalently, $z\bar{z} = 1$; in the complex plane this indeed looks like a circle. What is the "unit circle" in the dual number plane?

(A) All dual numbers where $a + b = 1$

(B) All dual numbers where $b = \pm 1$

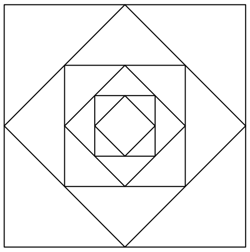
(C) All dual numbers where $a^2 = b^2$

$\boxed{\text{(D)}}$ All dual numbers where $a = \pm 1$

(E) All dual numbers where $ab = 1$

$\boxed{\mathcal{SOLN}}$ $(a + b\varepsilon)(a - b\varepsilon) = a^2$. In the dual plane this collection of points consists of two vertical lines at $a = \pm 1$. □

9. The figure shows only the first six of a sequence of squares. The outermost square is a unit square with area $1\,\text{cm}^2$. Each of the other squares is obtained by joining the midpoints of the sides of the squares before it. Find the sum of the infinite series of the perimeters (in cm) of all the squares.



(A) $4\sqrt{2}$

(B) $2(2-\sqrt{2})$

$\boxed{\text{(C)}}$ $8/(2-\sqrt{2})$

(D) $4$

(E) $8$

$\boxed{\mathcal{SOLN}}$ The largest square has a perimeter of 4. The sides of the second are $\sqrt{2}/2$ giving a perimeter of $4(\sqrt{2}/2)$. This leads to a geometric series $4+4(\sqrt{2}/2)+4(\sqrt{2}/2)^2+\cdots$. The series converges to $4/(1-\sqrt{2}/2)=8/(2-\sqrt{2})$. □

10. Suppose that $a$ and $b$ are real numbers and $f$ is differentiable at $x$. Express the limit

$$\lim_{h\to 0}\frac{f(ah+x)-f(bh+x)}{h}$$

in terms of $a, b$, and $f'(x)$.

(A) $\frac{1}{a}-\frac{1}{b}$

(B) $\frac{f'(x)}{a}-\frac{f'(x)}{b}$

(C) $f'(ax)+f'(bx)$

(D) $f'(ax)-f'(bx)$

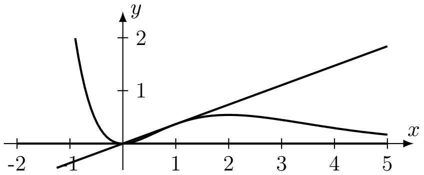
$\boxed{\text{(E)}}$ $af'(x)-bf'(x)$

$\boxed{\mathcal{SOLN}}$

$$\begin{aligned}&\lim_{h\to 0}\frac{f(ah+x)-f(bh+x)}{h}\\ &=\lim_{h\to 0}\frac{1}{h}(f(ah+x)-f(x)-f(bh+x)+f(x))\\ &=\lim_{h\to 0}\frac{f(ah+x)-f(x)}{h}-\lim_{h\to 0}\frac{f(bh+x)-f(x)}{h}\\ &=a\lim_{h\to 0}\frac{f(ah+x)-f(x)}{ah}-b\lim_{h\to 0}\frac{f(bh+x)-f(x)}{bh}\end{aligned}$$

□

11. For which $x$ value (other than 0) does the graph of $y=x^2\mathrm{e}^{-x}$ have a tangent line with a $y$-intercept of 0?



$\boxed{\text{(A)}}$ 1

(B) 2

(C) $\mathrm{e}-1$

(D) $\mathrm{e}$

(E) $\mathrm{e}+1$

$\boxed{\mathcal{SOLN}}$ The tangent line will pass through the origin if $f'(x)=\frac{f(x)}{x}$. This gives

$$\begin{aligned}xe^{-x}(2-x)&=\frac{x^2e^{-x}}{x}\\ 2x-x^2&=x\\ x&=0,1\end{aligned}$$

□

12. Solve the equation for $x$.

$$x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}=9$$

(A) $x=3$

(B) $x=4$

(C) $x=5$

$\boxed{\text{(D)}}$ $x=6$

(E) $x=7$

$\boxed{\mathcal{SOLN}}$

$$\begin{aligned}x+\sqrt{\underbrace{x+\sqrt{x+\sqrt{x+\cdots}}}_{9}}&=9\\ x+\sqrt{9}&=9\\ x+3&=9\\ x&=6\end{aligned}$$

□

13. Solve for $s$.

$$\sqrt{s\sqrt{n\sqrt{o\sqrt{w}}}} = 2$$

$$s > 0, \quad s = n = o = w$$

$\boxed{\text{(A)}}$ $s = 2\sqrt[15]{2}$

(B) $s = \sqrt[16]{2}$

(C) $s = 2^{16}\sqrt{2}$

(D) $s = 2^8$

(E) $s = 2\sqrt[16]{2}$

$\boxed{\mathcal{SOLN}}$

$$\sqrt{s\sqrt{n\sqrt{o\sqrt{w}}}} = \sqrt{s\sqrt{s\sqrt{s\sqrt{s}}}}$$
$$= \sqrt{s}\sqrt[4]{s}\sqrt[8]{s}\sqrt[16]{s} = s^{1/2}s^{1/4}s^{1/8}s^{1/16}$$
$$= s^{15/16} = 2$$

$\square$

14. Which is **NOT** equal to 2025?

(A) $1^3 + 2^3 + 3^3 + \ldots + 9^3$

(B) $45^2$

(C) base two (binary): 11111101001

$\boxed{\text{(D)}}$ $1^2 + 2^2 + 3^2 \ldots + 18^2$

(E) base five: 31100

$\boxed{\mathcal{SOLN}}$ 2025 is a perfect square: $45^2 = 2025$. Using summation formulas we find that $\sum_{n=1}^{9} n^3 = \left(\frac{9\cdot(9+1)}{2}\right)^2 = 2025$, but $\sum_{n=1}^{18} n^2 = \frac{18\cdot(18+1)\cdot(2\cdot18+1)}{6} = 2109$. $\square$

15. Suppose that Ali has three investment opportunities, where she magically knows the future performance:

- Option A: gain 10% in year 1, lose 10% in year 2, gain 10% in year 3.
- Option B: gain 5% in year 1, gain 4% in year 2, no change in year 3.
- Option C: gain 20% in year 1, lose 5% in year 2, lose 5% in year 3.

If Ali wants to end up with the most money, which investment should she choose?

(A) Option A only

$\boxed{\text{(B)}}$ Option B only

(C) Option C only

(D) Option A or C

(E) All options are equal.

$\boxed{\mathcal{SOLN}}$ The key is to not treat percentage changes as things that can be arithmetically added — if you do you might judge that A and C look the same. Suppose we start with \$100. Option A would yield \$110, \$99, \$108.9. Option B would yield: \$105, \$109.2, \$109.2. Option C would yeild \$120, \$114, \$108.3. Note that if there is a percentage gain followed by a percentage loss of the same percentage, the loss hits harder as it is taking a bite of a bigger number. $\square$

16. Suppose that $f$ is a function with domain $D = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$. Suppose that the outputs of the function $f(x)$ are *chosen* from the set $C = \{7, 8, 9\}$. Suppose that $A$ is a subset of $C$. Define $f^{-1}(A)$, called the *preimage* of $A$, to be the set $\{x\ :\ f(x) \in A\}$. If $f^{-1}(\{7\}) = \{1,\ 2\}$ and $f^{-1}(\{8\}) = \{3,\ 5\}$, then compute:

$$f^{-1}(\{f(4)\})$$

(A) $\{4\}$

(B) $\{1, 2\}$

(C) $\{3, 5\}$

(D) $\{6\}$

$\boxed{\text{(E)}}$ $\{4, 6\}$

$\boxed{\mathcal{SOLN}}$ We have already indicated everything that goes to 7 and 8 so that 4 must go to 9. The same is true for 6. □

17. What is the area of the largest rectangle that can be inscribed in a closed semicircle of radius 4?

(A) 32

$\boxed{\text{(B)}}$ 16

(C) $8\sqrt{2}$

(D) $16\sqrt{2}$

(E) $12\sqrt{3}$

$\boxed{\mathcal{SOLN}}$ Make a complete circle and notice the largest area rectangle inscribed in the circle is a square of side length $4\sqrt{2}$. This square has area 32 so the semicircle rectangle will have area 16. □

18. Jug A has 1 L of lemonade and jug B has 1 L of fruit punch. Pour $x$ mL of the lemonade from A into B, mix well, and then pour $x$ mL of the mixture from B back into A so each jug again contains 1 L of beverage.
Will the amount of fruit punch in A be more than, less than, or equal to the amount of lemonade in B?

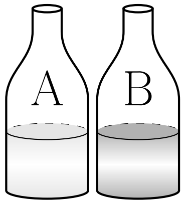
(A) more than

(B) less than

$\boxed{\text{(C)}}$ equal to

(D) depends on $x$

(E) not enough information



$\boxed{\mathcal{SOLN}}$ Algebra: All units are mL. After the first pour, B contains $x$ of lemonade and 1000 of punch for a total of $x+1000$, the fraction of which is punch is $\frac{1000}{x+1000}$. After pouring $x$ of the mixture back into A, the punch content of A is $x\left(\frac{1000}{x+1000}\right)$. Now the lemonade content of B is the total liquid in B minus the $x$ poured back, multiplied by the fraction of B that was lemonade: $(x + 1000 - x)\left(\frac{x}{x+1000}\right)$.
Logic: Since the two jugs end up with the same amount of liquid, however much lemonade ends up in B must be balanced by an equal amount of punch ending up in A. □

19. Which polar equation best represents the graph for $0 \leq \theta \leq 2\pi$?

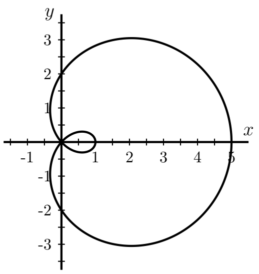
(A) $r = 3 - 2\sin\theta$

(B) $r = 2 + 3\sin\theta$

(C) $r = 3 + 2\sin\theta$

(D) $r = 3 + 2\cos\theta$

$\boxed{\text{(E)}}$ $r = 2 + 3\cos\theta$



$\boxed{\mathcal{SOLN}}$ Plot points for a few values of $\theta$ — start with $\theta = 0$. The two cosine choices are the only ones that give $x = 5,\ y = 0$. At $\theta = \pi$, $r = -1 \implies x = 1, y = 0$. □

20. Which pair of cubic *weird dice* produces the same probability distribution of sums as a pair of regular dice?

$\boxed{\text{(A)}}$ 1, 2, 2, 3, 3, 4 and 1, 3, 4, 5, 6, 8

(B) 1, 1, 3, 3, 3, 5 and 1, 2, 4, 6, 6, 7

(C) 1, 2, 2, 3, 3, 3 and 2, 3, 4, 5, 5, 9

(D) 0, 2, 2, 4, 4, 4 and 2, 3, 4, 5, 6, 7

(E) 1, 2, 2, 3, 4, 4 and 2, 3, 4, 5, 6, 8

$\boxed{\text{SOLN}}$

| | + | 1 | 2 | 2 | 3 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 3 | 4 | 4 | 5 |
| weird | 3 | 4 | 5 | 5 | 6 | 6 | 7 |
| | 4 | 5 | 6 | 6 | 7 | 7 | 8 |
| | 5 | 6 | 7 | 7 | 8 | 8 | 9 |
| | 6 | 7 | 8 | 8 | 9 | 9 | 10 |
| | 8 | 9 | 10 | 10 | 11 | 11 | 12 |

| | + | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| regular | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

□

21. What is the output of the following Python program?

```
(a, b) = (1, 1)
while a < 20:
  print(a)
  (a, b) = (b, a + b)
```

$\boxed{\text{(A)}}$ the Fibonacci numbers less than 20

(B) the squares of numbers less than 20

(C) the triangular numbers less than 20

(D) the counting numbers less than 20

(E) the prime numbers less than 20

$\boxed{\text{SOLN}}$ Manually run through the computational loop a couple of times. Doing so produces 1, 1, 2, 3, .... Variable values are not updated until a line break. □

22. For what value of $k$ is the line through the points $(3, 2k + 1)$ and $(8, 4k - 5)$ parallel to the $x$-axis?

(A) $-4$

(B) 2

(C) 0

(D) $-1$

$\boxed{\text{(E)}}$ 3

$\boxed{\text{SOLN}}$ The two points must have the same $y$-value. $2k + 1 = 4k - 5 \implies k = 3$ □

23. A highly composite number is a positive integer with more divisors (factors) than any other positive integer less than it. For example, 6 is highly composite because it has four divisors (1, 2, 3, 6) and no positive integer less than 6 has that many. What is the next highly composite number?

(A) 8

(B) 9

(C) 10

$\boxed{\text{(D)}}$ 12 (1, 2, 3, 4, 6, 12) = six divisors

(E) 24 (also highly composite)

$\boxed{\text{SOLN}}$ This means a base 12 number system would have been very convenient for fractions. Too bad we don't have six fingers on each hand. □

24. A factorial prime is a prime number that is one less or one more than a factorial. Which of the following is a factorial prime?

(A) 11 neither 10 nor 12 is a factorial

(B) 15 not prime

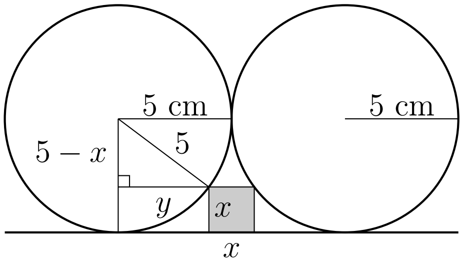
(C) 19 neither 18 nor 20 is a factorial

$\boxed{\text{(D)}}$ 23 $= 4! - 1$

(E) 119 $= 5! - 1$ but is not prime

$\boxed{\text{SOLN}}$ There are 27 factorial primes of the form $n! - 1$ and 24 of the form $n! + 1$ known as of October 2022. □

25. Two circles of radius 5 cm are tangent to the same line and tangent to each other. A square is inscribed between the circles and the line. What is the area of the square?



(A) $2\,\text{cm}^2$

(B) $3\,\text{cm}^2$

$\boxed{\text{(C)}}$ $4\,\text{cm}^2$

(D) $4.25\,\text{cm}^2$

(E) $5\,\text{cm}^2$

$\boxed{\mathcal{SOLN}}$ Draw two additional radii forming a right triangle. Label the dimensions of the square as $x$ and the lengths of the triangle sides as $5-x$ and $y$. The Pythagorean theorem gives $y^2+(5-x)^2=5^2$ or $y=\sqrt{10x-x^2}$. The value of $2y+x$ will equal twice the radius of the circles which is 10. Solve the equation $2\sqrt{10x-x^2}+x=10$ for $x$ by squaring both sides to get $4(10x-x^2)=(10-x)^2$. This gives a quadratic equation of $x^2-12x+20=0$ which has solutions $x=10, x=2$. The first solution is too large to fit the inscribed square. □

26. Find the solution to the following equation.

$$\left(\frac{1}{4}\right)^{x-4}=(\sqrt{2})^{6x-4}$$

(A) 5/2

$\boxed{\text{(B)}}$ 2

(C) 3

(D) 1/2

(E) 5

$\boxed{\mathcal{SOLN}}$ Rewrite 1/4 as $2^{-2}$ and $\sqrt{2}$ as $2^{1/2}$. Then multiply exponents.

$$\begin{aligned}(2^{-2})^{x-4}&=(2^{1/2})^{6x-4}\\2^{8-2x}&=2^{3x-2}\end{aligned}$$

Set the exponents equal to each other and solve. □

27. If the values of $a, b$, and $c$ are real, the solutions of $ax^2+bx+c=0$ may be rational, irrational, or complex ($\mathrm{i}=\sqrt{-1}$). In general, if $a, b$, and $c$ are complex, then the solutions will be complex. Find the solutions to the following quadratic equation.

$$x^2+\mathrm{i}x+2=0$$

(A) $\pm\mathrm{i}$

$\boxed{\text{(B)}}$ $\mathrm{i}, -2\mathrm{i}$

(C) $2\mathrm{i}, -\mathrm{i}$

(D) $\pm 2\mathrm{i}$

(E) $\mathrm{i}, 2\mathrm{i}$

$\boxed{\mathcal{SOLN}}$ Using the quadratic formula with $a=1$, $b=\mathrm{i}$, $c=2$ gives $x=\frac{-\mathrm{i}\pm\sqrt{\mathrm{i}^2-4(1)(2)}}{2(1)}$ which simplifies to $x=\frac{-\mathrm{i}\pm\sqrt{-9}}{2}=\frac{-\mathrm{i}\pm 3\mathrm{i}}{2}$. The solution set is $\{2\mathrm{i}/2, -4\mathrm{i}/2\}$. Can also be solved (quicker) by factoring: $(x+2\mathrm{i})(x-\mathrm{i})=0$ □

28. Simplify the expression.

$$\sin\left[\tan^{-1}\left(\frac{3}{4}\right)+\cos^{-1}\left(\frac{5}{13}\right)\right]$$

(A) 8/17

(B) 43/65

(C) 15/52

(D) 17/52

$\boxed{\text{(E)}}$ 63/65

$\boxed{\text{SOLN}}$ $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ and $\beta = \cos^{-1}\left(\frac{5}{13}\right)$. Sine angle sum identity: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$. Drawing right triangles with acute angles of $\alpha$ and $\beta$ gives us $\sin\alpha = 3/5, \sin\beta = 12/13$, and $\cos\alpha = 4/5$. □

29. Solve the equation for $n$.

$$\frac{n}{(n-1)!} - \frac{13}{n!} = \frac{n-1}{n!}$$

(A) 2

(B) 3

$\boxed{\text{(C)}}$ 4

(D) 5

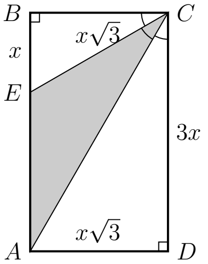
(E) 24

$\boxed{\text{SOLN}}$ Multiply the first fraction by $n$ over $n$ to get a common denominator of $n!$.

$$\frac{n^2}{n!} - \frac{13}{n!} = \frac{n-1}{n!}.$$

Multiply each term by $n!$ to get $n^2 - 13 = n - 1$ or $n^2 - n - 12 = 0$. The solution set to this is $\{-3, 4\}$ and only $n = 4$ solves the original equation. □

30. $ABCD$ is a rectangle. The three measured angles at vertex $C$ are all congruent. What fraction of the rectangle is shaded?



(A) $\sqrt{3}/4$

$\boxed{\text{(B)}}$ 1/3

(C) 1/2

(D) $\sqrt{3}/5$

(E) 1/4

$\boxed{\text{SOLN}}$ The three congruent angles at $C$ are all 30°. Each of the non-shaded triangles are $30° - 60° - 90°$ triangles. Label point $E$ and segment $BE$ as a length of $x$. Then $BC$ and $AD$ measure $x\sqrt{3}$, and $CD$ measures $3x$. From this we get $EA$ measures $2x$ and the shaded area out of the total area will be

$$\frac{(1/2)(2x)(x\sqrt{3})}{(3x)(x\sqrt{3})} = \frac{1}{3}$$ □

31. The general product of $2 \times 2$ matrices is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}.$$

The Hadamard (or Schur) product of matrices is found by multiplying components element-wise as illustrated below.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae & bf \\ cg & dh \end{bmatrix}$$

Which of the following is NOT a necessary condition for the general product to be equal to the Hadamard product?

(A) $bg = 0$

$\boxed{\text{(B)}}$ $g = \frac{ce}{d-c}$

(C) $ce + dg = cg$

(D) $cf = 0$

(E) $f = \frac{bh}{b-a}$

$\boxed{\text{SOLN}}$ Equate the right side of both products. We see that $bg = 0$ and $cf = 0$ are both necessary for the main diagonal. The bottom left entry of each requires $ce + dg = cg$. Solving this equation for $g$ gives

$$g = ce/(c - d)$$

which is the opposite of the correct choice. Solving the upper right entries' equation for $f$ gives $f = bh/(b - a)$. □

32. There are 396 men, women, and children in a room. If the ratio of women to men is 2:3, and the ratio of men to children is 1:2, how many men are in the room?

(A) 79

(B) 82

(C) 86

(D) 95

$\boxed{\text{(E)}}$ 108

$\boxed{\text{SOLN}}$

$$m + w + c = 396$$

$$\frac{w}{m} = \frac{2}{3} \implies w = \frac{2}{3}m$$

$$\frac{m}{c} = \frac{1}{2} \implies c = 2m$$

$$m + \frac{2}{3}m + 2m = 396$$

$$\frac{11}{3}m = 396 \implies m = 108$$

□

33. Mick's current age is double what Ruth's age was 35 years ago. 20 years ago, Mick's age was half of Ruth's current age. What is the difference between the current ages of Ruth and Mick?

(A) 5

$\boxed{\text{(B)}}$ 10

(C) 15

(D) 20

(E) 24

$\boxed{\text{SOLN}}$ Use $M$ and $R$ for current ages. Solve the system of equations:

$$\begin{cases} M = 2(R - 35) \\ M - 20 = \frac{1}{2}R \end{cases}$$

The solution is $M = 50$ and $R = 60$. □

34. If $A = 1001011_{\text{two}}$, $B = 33_{\text{four}}$, and $C = 12_{\text{eight}}$, what is $A \div B \times C$ in base ten?

$\boxed{\text{(A)}}$ 50

(B) 364004

(C) 0.5

(D) 14

(E) 35

$\boxed{\mathcal{SOLN}}$ $A = 1001010_{\text{two}} = 75_{\text{ten}}$, $B = 33_{\text{four}} = 15_{\text{ten}}$, and $C = 12_{\text{eight}} = 10_{\text{ten}}$. $A \div B \times C = 75 \div 15 \times 10 = 50$ □

35. In the Fibonacci sequence each number is the sum of the two preceding numbers, with the first two both 1.
The sum of the first and third of three consecutive Fibonacci numbers is three more than twice the second. Three times the first is eleven more than the second. What is the sum of the three numbers?

(A) 16

(B) 26

$\boxed{\text{(C)}}$ 42

(D) 68

(E) 110

$\boxed{\mathcal{SOLN}}$ Use $x$ and $y$ for the first two; the third is $x + y$. Solve the system:

$$\begin{cases} x + (x + y) = 3 + 2y \\ 3x = 11 + y \end{cases}$$

The solution is $x = 8$ and $y = 13$.
$8 + 13 + (8 + 13) = 42$. □

36. Napoleon needs to load Tina, his 300-pound pet llama, into a truck to take her to the vet for a checkup. The ramp to the truck has an angle of elevation of 30°. How much force is required to pull Tina up the frictionless ramp leading into the truck?

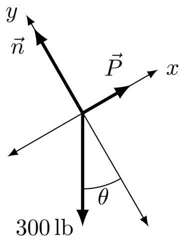
(A) 130 pounds

$\boxed{\text{(B)}}$ 150 pounds

(C) 173 pounds

(D) 200 pounds

(E) 260 pounds



$\boxed{\mathcal{SOLN}}$ $\vec{P}$ is the pulling force,
$\vec{n}$ is the normal force.

$$\sum F_x = P - (300\,\text{lb})(\sin 30°) = 0$$

$$P = (300\,\text{lb})(\sin 30°) = (300\,\text{lb})(\tfrac{1}{2})$$ □

37. $\overline{AB}$ is parallel to $\overline{CD}$.
$AC = 6\,\text{ft}$, $CE = 18\,\text{ft}$, and $CD = 27\,\text{ft}$.
What is the length of $\overline{AB}$?

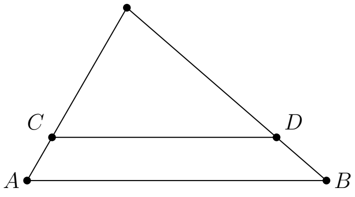
(A) 9 ft

(B) 20 ft

(C) 33 ft

$\boxed{\text{(D)}}$ 36 ft

(E) 54 ft



$\boxed{\mathcal{SOLN}}$

$$\triangle CED \sim \triangle AEB \implies \frac{AE}{CE} = \frac{AB}{CD}$$

$$\frac{24}{18} = \frac{AB}{27} \implies AB = \frac{24 \cdot 27}{18}$$ □

38. It is 2025 and Dan is an adult who has had his birthday this year. Dan noticed that his age is equal to the sum of the digits in his birth year. What is the sum of the digits in his age?

(A) 7
$\boxed{\text{(B)}}$ 9
(C) 12
(D) 14
(E) 15

$\boxed{\mathcal{SOLN}}$ Let the birth year be $19xy$. Sum of digits of birth year $= 1 + 9 + x + y = 10 + x + y$. Age $= 2025 - (1900 + 10x + y) = 125 - 10x - y$. Setting this equal to the sum of digits and arithmetic yields $115 = 11x + 2y$. Since $x$ and $y$ are each whole numbers less than 10, $y$ is at most 9. Therefore $11x$ is at least 97, meaning $x = 9$ and then $y$ must be 8. So age $=$ 27 and sum of digits in age $=$ 9. Note: if birth year is $20xy$ the birth year is 2016 or 2007 and age is 9 or 18 respectively and the sum of digits is still 9. □

39. Let $M$ be the **sum of the solutions** to $e^{-x}\sin x - e^{-x}\cos x = 0$ in $[0, 2\pi]$. Find $\csc M$.

$\boxed{\text{(A)}}$ $-1$
(B) $-\frac{\sqrt{3}}{2}$
(C) 1
(D) $\frac{2}{\sqrt{3}}$
(E) 2

$\boxed{\mathcal{SOLN}}$

$$e^{-x}(\sin x - \cos x) = 0$$
$$\sin x - \cos x = 0$$
$$\sin x = \cos x$$
$$x = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$$
$$M = \frac{\pi}{4} + \frac{5\pi}{4} = \frac{3\pi}{2}$$
$$\csc M = \csc\frac{3\pi}{2} = -1$$

□

40. Solve. $\log_5(\log_2 x) = 1$

(A) 1
(B) 7
(C) 10
(D) 25
$\boxed{\text{(E)}}$ 32

$\boxed{\mathcal{SOLN}}$

$$\log_5(\log_2 x) = 1$$
$$5^1 = \log_2 x$$
$$2^5 = x$$
$$x = 32$$

□