

# Snow College Mathematics Contest

April 1, 2008

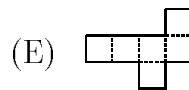
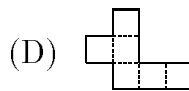
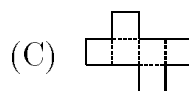
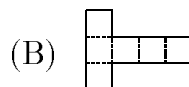
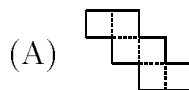
Senior division: grades 10-12

Form: **T**

Bubble in the single best choice for each question you choose to answer.

1. What is the contrapositive of “If 2 is even, then 4 is even.”?  
(A) “If 2 is odd, then 4 is odd.”  
(B) “If 4 is even, then 2 is even.”  
(C) “If 4 is even, then 2 is odd.”  
(D) “If 4 is odd, then 2 is odd.”  
(E) “If 2 is even, then 4 is odd.”
2. Given that  $a = 1/x$ ,  $b = 9a$ ,  $c = 1/b$ ,  $d = 9c$ ,  $e = 1/d$ , and  $a$ ,  $b$ ,  $c$ , and  $d$  are all distinct non-zero numbers, then which must equal  $x$ ?  
(A)  $a$   
(B)  $b$   
(C)  $c$   
(D)  $d$   
(E)  $e$

4. Which of the following patterns below CANNOT be folded along the dashed lines to form a cube?



3. What is the minimum value of the function  $f(x) = 4x^2 - 8x + 15$ ?  
(A) 1  
(B) 11  
(C) 15  
(D) 27  
(E) None of the above

5. Simplify  $(\sqrt{2})^{\log_2 9}$ .  
(A) 3  
(B) 4.5  
(C) 6  
(D) 7.5  
(E) 9

6. A closed operational system  $(G, *)$  is a *group* iff (if and only if)

- (i)  $(a * b) * c = a * (b * c)$  for every  $a, b, c \in G$ ; [Associativity]
- (ii) There exists an element  $e \in G$  such that  $e * a = a$  and  $a * e = a$  for every  $a \in G$ ; [Identity Element]
- (iii) For each  $a \in G$ , there exists an element  $a^{-1}$  such that  $a * a^{-1} = e$  and  $a^{-1} * a = e$ . [Inverse Element]

Is the operational system below a group?

$*$	$\alpha$	$\beta$	$\gamma$
$\alpha$	$\alpha$	$\beta$	$\gamma$
$\beta$	$\beta$	$\alpha$	$\alpha$
$\gamma$	$\gamma$	$\alpha$	$\beta$

- (A) Yes
- (B) No, because (i) is violated.
- (C) No, because (ii) is violated.
- (D) No, because (iii) is violated.
- (E) No, because two are violated.

7. Ed bought clothes for school. In the first store he spent half his money plus \$10. In the second store he spent half of what was left plus \$5. In the last store he spent three-fourths of what was left and came home with \$5. How much did he start out with?

- (A) \$120
- (B) \$80
- (C) \$60
- (D) \$40
- (E) \$30

8. Seven friends are sitting in a theater watching a show. The row they are all in contains exactly seven seats. After intermission, they return to the same row but choose seats randomly. What is the probability that neither of the people sitting in the two aisle seats was previously sitting in an aisle seat?

- (A)  $3/7$
- (B)  $10/21$
- (C)  $11/21$
- (D)  $4/7$
- (E)  $25/49$

9. A square has four corners (or vertices), four edges, and one face. A cube has eight corners, twelve edges, six faces, and one volume. How many faces would a 4-D hypercube (*a.k.a.*, a tesseract) have?

- (A) 24
- (B) 28
- (C) 30
- (D) 32
- (E) 36

10. How many times does a clock strike in one day if it strikes the time on each hour and once on each half hour? (e.g., 2:00 = two dings, 2:30 = one ding, 3:00 = three dings, 3:30 = one ding.)

- (A) 24
- (B) 48
- (C) 90
- (D) 180
- (E) 210

11. How many squares (of any size from  $1 \times 1$  to  $8 \times 8$ ) are there on a checker board (an  $8 \times 8$  grid)?

- (A) 40320
- (B) 64
- (C) 204
- (D) 85
- (E) 144

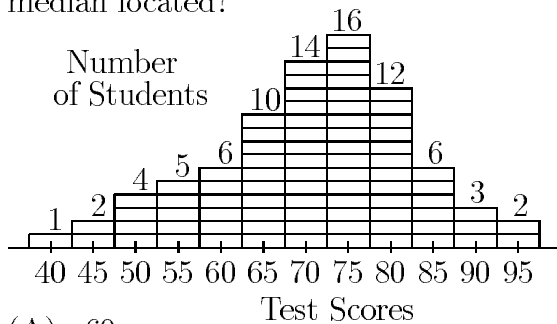
12. If  $\sin(\theta) = 0.5$  then which of the following could be the value of  $\cos(\theta)$ ?

- (A) 0.5
- (B) 0
- (C) -0.5
- (D)  $\sqrt{3}/2$
- (E)  $\sqrt{2}/2$

13. Simplify  $2^{2008} + 2^{2008}$ .

- (A)  $4^{2008}$
- (B)  $4^{4016}$
- (C)  $2^{2009}$
- (D)  $2^{4016}$
- (E) None of these

14. Consider this histogram of scores for 81 students taking a test. In which interval is the median located?



- (A) 60
- (B) 65
- (C) 70
- (D) 75
- (E) 80

15. The vector cross product is defined as

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

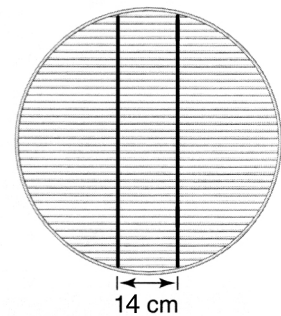
What is  $(3\hat{i} + 4\hat{j} - 5\hat{k}) \times (1\hat{i} + 2\hat{j} - 1\hat{k})$ ?

- (A)  $(-6\hat{i} - 2\hat{j} - 2\hat{k})$
- (B)  $(6\hat{i} - 2\hat{j} - 2\hat{k})$
- (C)  $(6\hat{i} + 2\hat{j} + 2\hat{k})$
- (D)  $(-6\hat{i} - 2\hat{j} + 2\hat{k})$
- (E)  $(6\hat{i} - 2\hat{j} + 2\hat{k})$

16. Consider  $f(x) = x^3 - x + c$ . When  $c = 0$  there are three real roots (you can verify this by setting  $c = 0$  and factoring the left side of  $x^3 - x = 0$ ). Characterize the roots when  $c = 1$ .

- (A) two real; one complex
- (B) one real; two complex
- (C) three real
- (D) three complex
- (E) None of the above

17. In the BBQ grill (diameter 50 cm) pictured, the two parallel support rods are equidistant from the center. What is the length of one of them?



- (A) 46 cm
- (B) 47 cm
- (C) 48 cm
- (D) 49 cm
- (E) 50 cm

18. A community group has 500 people. At the April 1 dance, new members pay only \$14, whereas longtime members pay \$20. As a result, all of the new members attend, but only 70% of the longtime members attend. How much revenue is collected?

- (A) \$7000
- (B) \$6000
- (C) \$5800
- (D) \$5400
- (E) There is not enough information.

19. Given the scoring formula  $\text{Score} = 4R - W$ , where  $R$  and  $W$  are the number right and wrong respectively, and not all questions must be answered, how many different integers between  $-40$  and  $160$  are not possible scores on this test?

- (A) None; all are possible scores
- (B) 1, 2, or 3
- (C) 4, 5, or 6
- (D) 7, 8, or 9
- (E) More than 9

20. Call  $P_1$  the probability of getting a sum of 7 when two fair dice are thrown. Call  $P_2$  the probability of getting a sum of 7 when a fair die is thrown and a dial spun on a spinner with four equal sections numbered 1, 2, 3, and 4. Which statement is true?

- (A)  $P_1 = P_2$
- (B)  $P_1 < P_2$
- (C)  $P_1 > P_2$
- (D)  $P_1 + P_2 > 1$
- (E) There is not enough information.

21. What is the solution set of  $|2x - 3| < \frac{1}{2}$ ?

- (A)  $\{x \mid x < \frac{5}{4}\} \cup \{x \mid x > \frac{7}{4}\}$
- (B)  $\{x \mid x > \frac{5}{4}\} \cap \{x \mid x < \frac{7}{4}\}$
- (C)  $\{x \mid x \leq \frac{5}{4}\} \cap \{x \mid x \geq \frac{7}{4}\}$
- (D)  $\{x \mid x > \frac{5}{4}\} \cup \{x \mid x < \frac{7}{4}\}$
- (E) None of these

22. What is the output of the following computer program?

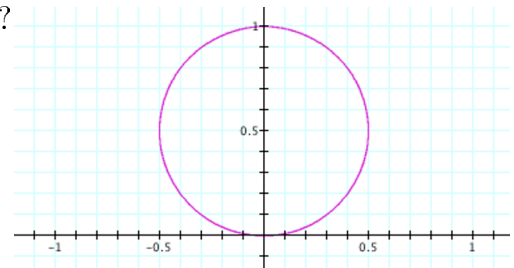
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10 for i = 1 to 3
20 for j = 1 to 3
30 if i < j then print i:
    else print j
40 next j
50 next i

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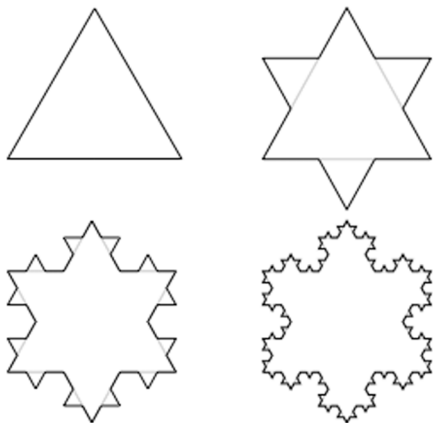
- (A) 1, 1, 1, 1, 2, 2, 1, 2, 3
- (B) 1, 2, 3, 2, 2, 3, 3, 3, 3
- (C) 1, 2, 2, 3, 3, 3
- (D) 1, 2, 3, 3, 2, 2, 2, 1, 1
- (E) 1, 2, 3, 1, 2, 3, 1, 2, 3

23. Which polar equation best represents the graph?



- (A)  $r = \theta$
- (B)  $r = \sin \theta$
- (C)  $r = \cos \theta$
- (D)  $r = 1$
- (E)  $\theta = 2\pi$

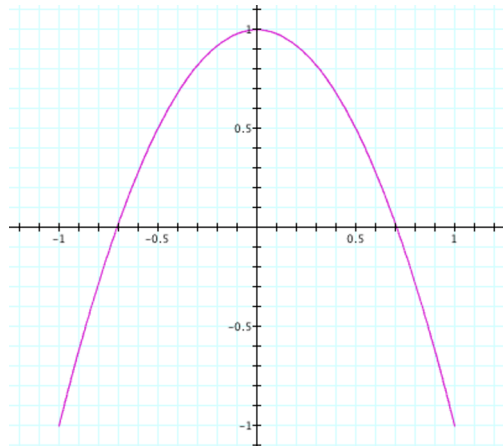
24. The Koch snowflake is constructed by starting with an equilateral triangle with sides of length 1. Then construct an equilateral triangle on the middle third of each side and erase the base of each of the new triangles. Repeat indefinitely.



The original perimeter is 3. After the first iteration the perimeter is 4. What is the perimeter after  $n$  iterations?

- (A)  $3n$   
 (B)  $\frac{4}{3}n$   
 (C)  $3(\frac{4}{3}n)$   
 (D)  $3(\frac{4}{3})^n$   
 (E)  $(3n)^{4/3}$

25. Which set of parametric equations will produce the graph shown?



- (A)  $\begin{cases} x(t) = \sin t \\ y(t) = \cos t \end{cases} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$   
 (B)  $\begin{cases} x(t) = t \\ y(t) = t^2 \end{cases} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$   
 (C)  $\begin{cases} x(t) = t \\ y(t) = \cos t \end{cases} \quad -1 \leq t \leq 1$   
 (D)  $\begin{cases} x(t) = \sin 2t \\ y(t) = \cos t \end{cases} \quad 0 \leq t \leq 2\pi$   
 (E)  $\begin{cases} x(t) = \sin t \\ y(t) = \cos 2t \end{cases} \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$

26. Given that  $\ln 1 = 0$ ,  $\ln 5 = 1.6094$ , and  $\ln 2 = 0.6931$ , find  $\ln 0.2$ .

- (A)  $-0.06931$   
 (B)  $1.0644$   
 (C)  $-1.6094$   
 (D)  $0.06931$   
 (E) None of these

27. Think of something so hot it glows, e.g., a branding iron. The power (energy/time) radiated by a blackbody per unit area is proportional to the fourth power of the absolute temperature.

$$\frac{P}{A} \propto T^4$$

By what factor is the power radiated from a given blackbody increased if its temperature is raised from 100 K to 300 K?

- (A) 3
- (B) 9
- (C) 27
- (D) 300
- (E) None of the above

28. All numbers in this problem are in base five. What is  $234 + 331$ ?

- (A) 565
- (B) 4302
- (C) 1120
- (D) 160
- (E) 3041

29. Ephraim has a plow that can clear 3 in of snow from the streets in 6 h. By using this plow and another plow from Manti at the same time, the same amount of snow can be cleared in 2.4 h. How long would it take the plow from Manti to do the job alone?

- (A) 1.8 h
- (B) 2.2 h
- (C) 3.6 h
- (D) 4.2 h
- (E) None of these

30. A cube has a volume of  $512 \text{ cm}^3$ . What is the surface area of the cube?

- (A)  $384 \text{ cm}^2$
- (B)  $256 \text{ cm}^2$
- (C)  $216 \text{ cm}^2$
- (D)  $96 \text{ cm}^2$
- (E) None of these

31. Upon investigating, the police found that one of a group of four girls had stolen a car. The police knew that three of the girls would always tell the truth, but that one consistently lied. When questioned, the girls made the following statements:

Alice: Betty did it.

Betty: Deb did it.

Carla: I didn't do it.

Deb: Betty lied when she said I did it.

Who stole the car?

- (A) Alice
- (B) Betty
- (C) Carla
- (D) Deb
- (E) Not enough information

32. The average of  $A$  and  $2B$  is 7, and the average of  $A$  and  $2C$  is 8. What is the average of  $A$ ,  $B$ , and  $C$ ?

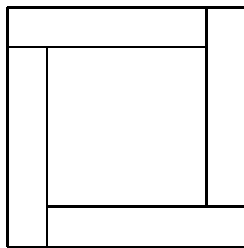
- (A) 3
- (B) 4
- (C) 5
- (D) 6
- (E) 9

33. Find the determinant:  $\begin{vmatrix} i & -i \\ -i & i \end{vmatrix}$

- (A)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$   
 (B) 0  
 (C) 1  
 (D) 2  
 (E) None of the above

34. A square is covered by a design made up of four identical rectangles surrounding a central square, as shown. If the area of the central square is  $\frac{4}{9}$  the area of the entire design, find the ratio of the length of a rectangle to the side of the central square.

- (A)  $\frac{5}{4}$   
 (B)  $\frac{4}{3}$   
 (C)  $\frac{7}{5}$   
 (D)  $\frac{3}{2}$   
 (E)  $\frac{8}{5}$



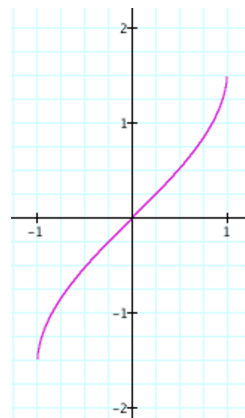
35. Matrix multiplication is not commutative in general. However, the following matrices  $A$  and  $B$  do commute. What is their product?

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \quad B = \begin{bmatrix} \cos \beta & \sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$$

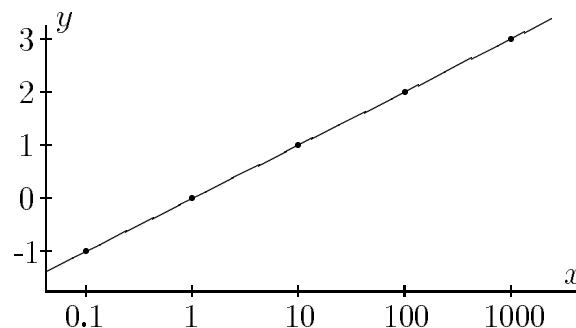
- (A)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$   
 (B)  $\begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha - \beta) \\ \sin(\alpha - \beta) & \cos(\alpha + \beta) \end{bmatrix}$   
 (C)  $\begin{bmatrix} \cos(\alpha - \beta) & \sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha - \beta) \end{bmatrix}$   
 (D)  $\begin{bmatrix} \sin(\alpha + \beta) & \cos(\alpha - \beta) \\ \cos(\alpha - \beta) & \sin(\alpha + \beta) \end{bmatrix}$   
 (E)  $\begin{bmatrix} \sin(\alpha - \beta) & \cos(\alpha + \beta) \\ \cos(\alpha + \beta) & \sin(\alpha - \beta) \end{bmatrix}$

36. Which equation best represents the graph?

- (A)  $y = \tan^{-1} x$   
 (B)  $y = \cos^{-1} x$   
 (C)  $y = \sec^{-1} x$   
 (D)  $y = \csc^{-1} x$   
 (E)  $y = \sin^{-1} x$



37. Which equation best represents the graph?



- (A)  $y = e^x$   
 (B)  $y = mx + b$   
 (C)  $y = 10^x$   
 (D)  $y = \log x$   
 (E)  $y = \frac{1}{2}x$

38. It's Sophie's birthday! Sophie Germain, famous, self-taught, woman mathematician, was born April 1, 1776. A prime number  $p$  is called a *Sophie Germain prime* if  $2p+1$  is also prime. All Sophie Germain primes  $p > 3$  are equivalent to \_\_\_\_\_ (mod 6).

- (A) 1  
 (B) 2  
 (C) 3  
 (D) 4  
 (E) 5

39. Add any integer  $n$  to the square of  $2n$  to produce an integer  $m$ . For how many values of  $n$  is  $m$  prime?
- (A) 0
  - (B) 1
  - (C) 2
  - (D) a finite number  $> 2$
  - (E)  $\infty$
40. A door is 4 ft wide and 7 ft tall. If the door is standing open at a  $90^\circ$  angle with the door frame, what is the greatest distance in feet from the outer top corner of the door to a point on the door frame?
- (A) 8
  - (B) 9
  - (C) 9.5
  - (D) 10
  - (E) 11