Snow College Mathematics Contest

April 7, 2009 **Senior division: grades 10-12** **Form: T**

Bubble in the single best choice for each question you choose to answer.

1. Simplify $7^{2009} \pmod 6$.
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

2. What are the asymptotes of $y = \tan x$?
   (A) $y = n\pi$ (n an integer)
   (B) $x = n\pi$ (n an integer)
   (C) $x = n\frac{\pi}{2}$ (n an integer)
   (D) $y = (2n+1)\frac{\pi}{2}$ (n an integer)
   (E) $x = (2n+1)\frac{\pi}{2}$ (n an integer)

3. If a steel ball one inch in diameter weighs one pound, how much will a steel ball two inches in diameter weigh?
   (A) 2 lbs
   (B) 3 lbs
   (C) 4 lbs
   (D) 5 lbs
   (E) 8 lbs

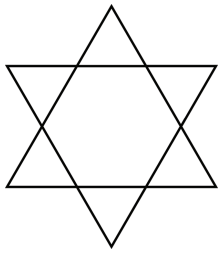
4. In a bowl of red, green, blue, and yellow jelly beans, all but 16 are red, all but 16 are green, all but 16 are blue, and all but 18 are yellow. How many jelly beans are there?
   (A) 16
   (B) 18
   (C) 22
   (D) 48
   (E) 66

5. A stock loses 60% of its value. What must the percent *increase* be to recover all of its lost value?
   (A) 60%
   (B) 120%
   (C) 150%
   (D) 200%
   (E) 250%

6. The sum $\cos 1° + \cos 2° + \cos 3° + \ldots + \cos 357° + \cos 358° + \cos 359°$ is equal to
   (A) $\pi/2$
   (B) π
   (C) 0
   (D) 1
   (E) -1

7. Over the life of a car the odometer rolls from 000 000 to 999 999. How many times does the digit 1 appear in the odometer reading? (Example: the reading 131 518 has three occurrences.)
   (A) 100 000
   (B) 111 111
   (C) 600 000
   (D) 666 666
   (E) 700 000

8. When two congruent equilateral triangles share a common center, their union can be a star. If their intersection is a regular hexagon with an area of 60, what is the area of one of the original equilateral triangles?

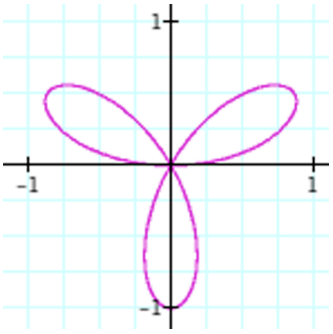


(A) 90
(B) 60
(C) 120
(D) 50
(E) 75

9. What is the best parabolic approximation for $\cos x$ near $x = 0$?

(A) x^2
(B) $-x^2 + 1$
(C) $x^2 + 1$
(D) $\frac{x^2}{2} + 1$
(E) $-\frac{x^2}{2} + 1$

10. Which polar equation best represents the graph?



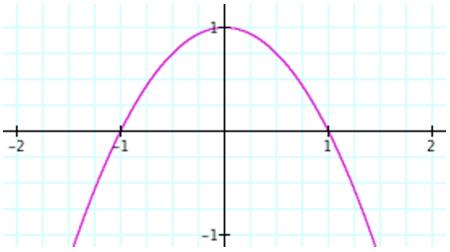
(A) $r = 3\theta$
(B) $r = \sin 3\theta$
(C) $r = \cos 3\theta$
(D) $r = 3\cos\theta$
(E) $r = \cos\theta$

11. Following the logic used in the first three squares, what is the missing number in the fourth square?

5 50 5	6 36 3
2	2
4 28 7	8 ? 2
1	3

(A) 48
(B) 64
(C) 4
(D) 12
(E) 40

12. What is the area bounded by the x-axis and the curve $y = -x^2 + 1$?



(A) 1
(B) $\frac{\pi}{3}$
(C) $\frac{4}{3}$
(D) $\frac{5}{3}$
(E) 2

13. An *equivalence relation* is a relation which is reflexive, symmetric, and transitive. Consider the relation $\mathcal{R}$:

$$x \mathcal{R} y \quad \text{if and only if} \quad x \geq y$$

Which statement is true?

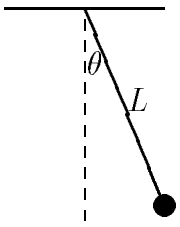
(A) $\mathcal{R}$ is an equivalence relation.
(B) $\mathcal{R}$ is not an equivalence relation only because it is not reflexive.
(C) $\mathcal{R}$ is not an equivalence relation only because it is not symmetric.
(D) $\mathcal{R}$ is not an equivalence relation only because it is not transitive.
(E) $\mathcal{R}$ is not an equivalence relation because two criteria fail.

14. The *eigenvalues* of a matrix A are the values of λ which satisfy $\det(\lambda I - A) = 0$. What are the eigenvalues of the following matrix?

$$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

(A) $2, -3$
(B) $4, -2$
(C) $3, 10$
(D) $-2, 5$
(E) $-3, -10$

15. The period T of a small-angle simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$ where g is the acceleration due to gravity and L is the length of the pendulum. By what factor does the period increase when the length is tripled?



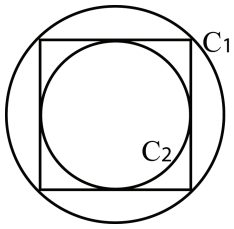
(A) 3
(B) $\sqrt{3}$
(C) $\sqrt{9.8}$
(D) 9
(E) $3\sqrt{2}$

16. A *fixed point* for a function $f(x)$ is a real number r such that $f(r) = r$. How many of the following classes of functions *must* have a fixed point?

- polynomial of odd degree > 1
- polynomial of even degree > 0
- trig function $y = A\sin Bx + C$
- rational function $y = (x - a)/(x - b)$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

17. A square is inscribed in a circle C_1. A second circle C_2 is inscribed in the square. What is the ratio of the areas of the circles, A_1/A_2?



(A) $\sqrt{2}$
(B) π
(C) $\pi/2$
(D) $\pi\sqrt{2}$
(E) 2

18. Which of the following is the identity function $f(x) = x$ for *all* real numbers?

(A) $f(x) = \mathrm{e}^{\ln x}$
(B) $f(x) = \ln \mathrm{e}^{x}$
(C) $f(x) = \sin(\arcsin x)$
(D) $f(x) = \arctan(\tan x)$
(E) $f(x) = \sqrt{x^2}$

19. A *perfect number* is a positive integer which is the sum of its proper positive divisors (that is, excluding the number itself). For example, 28 is a perfect number because $1 + 2 + 4 + 7 + 14 = 28$.
What is the smallest perfect number?

(A) 2
(B) 4
(C) 6
(D) 12
(E) 14

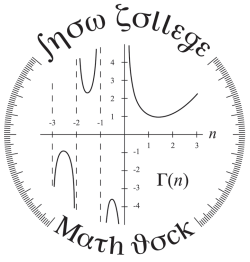
20. A general hypersphere in n dimensions is the locus of points equidistant r from a central point. In 2D space this is a circle whose circumference is $2\pi r$ and whose interior area is πr^2. In 3D space this is a sphere whose surface area is $4\pi r^2$ and whose interior volume is $\frac{4}{3}\pi r^3$. Find a pattern to deduce the hypersurface area of a 4D hypersphere given its interior hypervolume is $\frac{1}{2}\pi^2 r^4$.

(A) $2\pi^2 r^3$
(B) $8\pi^2 r^3$
(C) $8\pi r^3$
(D) $4\pi r^3$
(E) $2\pi r^3$

dim	surface	interior
1	2	$2r$
2	$2\pi r$	πr^2
3	$4\pi r^2$	$\frac{4}{3}\pi r^3$
4		$\frac{1}{2}\pi^2 r^4$
5	$\frac{8}{3}\pi^2 r^4$	$\frac{8}{15}\pi^2 r^5$

21. The Snow College Math Contest logo features the Gamma function which has the property $\Gamma(n+1) = n\Gamma(n)$ (for $n > 0$). Given $\Gamma(1) = 1$, what is $\Gamma(6)$?

(A) 24
(B) 30
(C) 60
(D) 120
(E) 720



22. Which of the following is NOT a factor of $x^4 - 4x^3 - x^2 + 16x - 12$?

(A) $x - 2$
(B) $x + 2$
(C) $x - 1$
(D) $x + 1$
(E) $x - 3$

23. A distance runner can run 6 mph downhill, but his speed is cut to 2 mph when he runs uphill. If he runs 6 miles downhill and 6 miles uphill, what is his average speed?

(A) 2 mph
(B) 3 mph
(C) 4 mph
(D) 5 mph
(E) 6 mph

24. The numbers $1, a, 9$ form an arithmetic sequence. The numbers $1, b, 9$ form a geometric sequence. What is $a + b$?

(A) 1
(B) 6
(C) 8
(D) 9
(E) 11

25. What is the area of the largest triangle that can be inscribed in a semi-circle of radius 3?

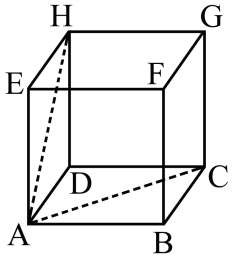
(A) 9
(B) 9.5
(C) 18
(D) 27
(E) 54

26. Simplify the inequality.

$$1 < \frac{1}{3 - 2x} < 3$$

(A) $-\frac{3}{4} < x < -\frac{3}{16}$
(B) $\frac{3}{4} < x < 3$
(C) $-\frac{16}{3} < x < -\frac{4}{3}$
(D) $1 < x < \frac{4}{3}$
(E) $-\frac{4}{3} < x < 1$

27. Diagonals AC and AH are drawn in cube ABCDEFGH. What is the measure of $\angle$ CAH?



(A) 60°
(B) 72°
(C) 45°
(D) 90°
(E) 54°

28. Find the value of x in the continued fraction.

$$\cfrac{1}{x - \cfrac{1}{x - \cfrac{1}{x - \cfrac{1}{x - \cdots}}}} = 2$$

(A) 0.5
(B) 1
(C) 1.5
(D) 2
(E) 2.5

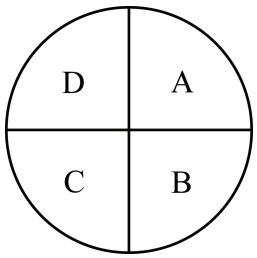
29. Two standard dice are rolled twice. What is the probability of obtaining an even sum followed by another even sum?

(A) 0.25
(B) 0.45
(C) 0.5
(D) 0.75
(E) 0.866

30. What are the sum and product of the roots of $x^2 + 3x - 2 = 0$?

(A) 3 and 2
(B) 3 and -2
(C) -3 and 2
(D) -3 and -2
(E) None of these

31. Gardens A and B each contain 50 plants. The average number of plants in Gardens B and C is 65. Which of the following is true if x represents the average number of plants in Gardens C and D?



(A) $x < 45$
(B) $x = 45$
(C) $45 < x < 46$
(D) $x \geq 46$
(E) Not enough information

32. How long is an escalator if it makes a $30°$ angle with the floor and caries people a vertical distance of 20 ft between floors?

(A) 10 ft
(B) 20 ft
(C) 30 ft
(D) 40 ft
(E) 50 ft

33. Pascal's triangle has 1s at each end of each row and the other numbers being the sum of the two immediately diagonally above it.

				1						row 0
			1		1					row 1
		1		2		1				row 2
	1		3		3		1			row 3
1		4		6		4		1		row 4
				⋮						

What is the sum of the numbers in row 7?

(A) 64
(B) 65
(C) 66
(D) 67
(E) 128

34. If p is a prime number and $p^3 + 5$ is also prime, then $p^5 + 7$ is

(A) prime.
(B) composite.
(C) either prime or composite.
(D) neither prime nor composite.
(E) Not enough information

35. Simplify $\sqrt{1 - \cos^2 19°} - \sqrt{1 - \sin^2 98°}$.

(A) $\sin 19° - \cos 98°$
(B) $-\sin 19° - \cos 98°$
(C) $-\sin 19° + \cos 98°$
(D) $\sin 19° + \cos 98°$
(E) $-\sin 98° + \cos 19°$

36. Find the median for the set of values.

3, 13, 4, 1, 4, 6, 7, 1, 5, 1

(A) 1
(B) 3
(C) 4
(D) 4.5
(E) 5

37. Of the properties commutative, associative, and identity, which hold for the operator $\otimes$ given in the table?

$\otimes$	a	b	c
a	a	b	c
b	b	a	b
c	c	b	a

(A) commutative and identity only

(B) commutative and associative only

(C) identity and associative only

(D) commutative only

(E) all three

38. Which statement about quadrilaterals is NOT correct?

(A) All rhombuses are parallelograms.

(B) All trapezoids are parallelograms.

(C) All squares are rhombuses.

(D) All squares are rectangles.

(E) All rectangles are parallelograms.

39. The repeating decimal $0.\overline{63} = 0.636363\ldots$ can be represented as a rational number p/q where p and q are relatively prime integers. What is $p+q$?

(A) 16

(B) 18

(C) 21

(D) 63

(E) 77

40. What is the inverse of the matrix?

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

(A) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$

(B) $\frac{1}{3}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

(C) $\frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

(D) $\frac{1}{3}\begin{bmatrix} -1 & 2 \\ -1 & -1 \end{bmatrix}$

(E) $\frac{1}{3}\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$